On the cohomology of Hecke congruence subgroup

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Topology Students Workshop
June 8-12, 2026

Motivation & Background

Cohomology of $\mathrm{SL}_n(\mathbb{Z})$

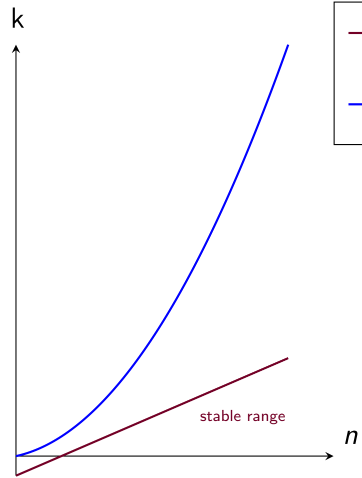

Fig 1: $\mathrm{H}^k(\mathrm{SL}_n(\mathbb{Z}); \mathbb{Q})$

— $k = n - 2$ Borel 1974, Li–Sun 2015

— Lee–Szczarba 1976
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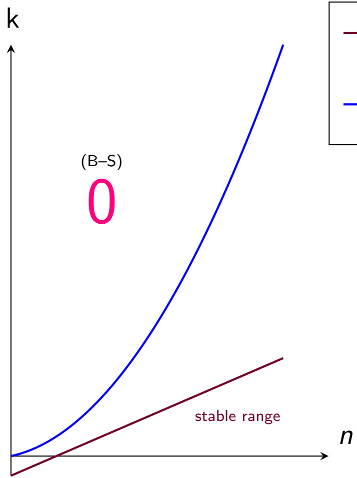

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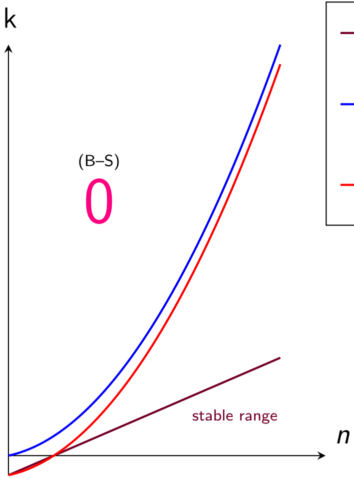

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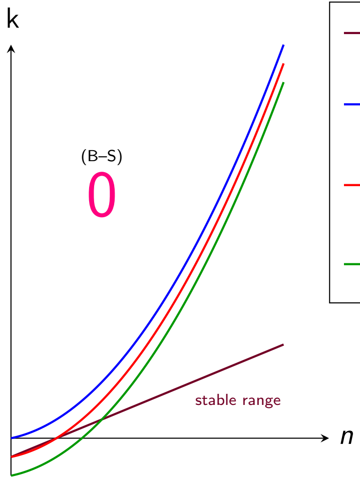

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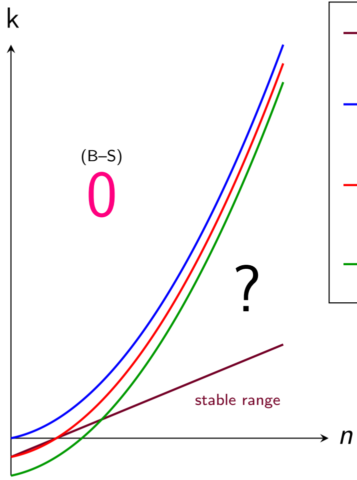

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Conjecture (Church–Farb–Putman 2014):

$$\mathrm{H}^{\binom{n}{2}-k}(\mathrm{SL}_n(\mathbb{Z});\mathbb{Q}) = 0 \text{ for } k \leq n-2.$$

Cohomology of $\Gamma_n(p)$

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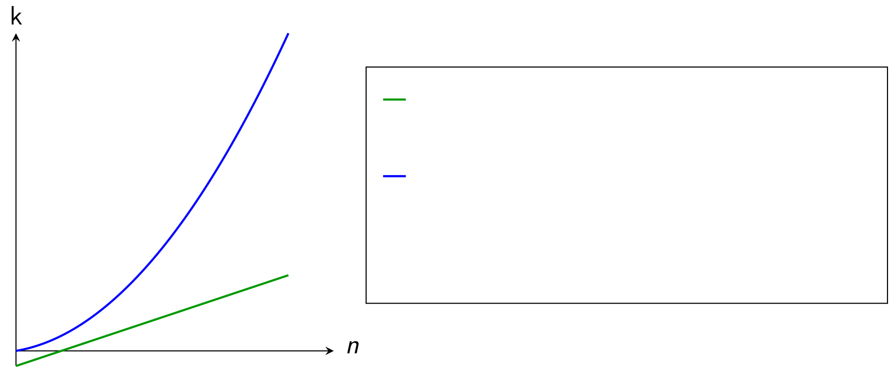

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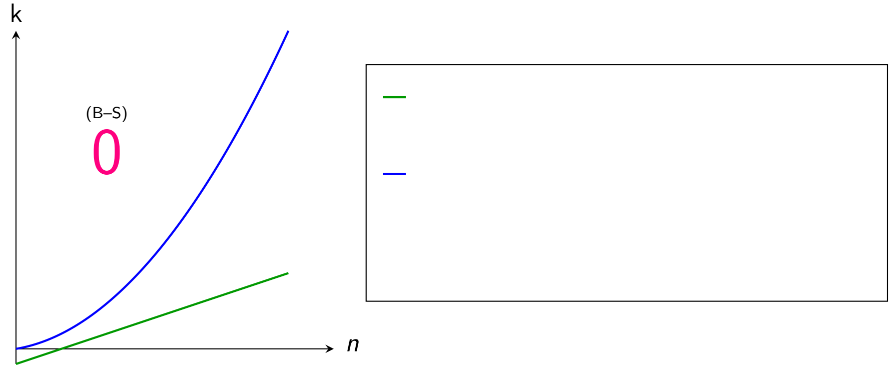

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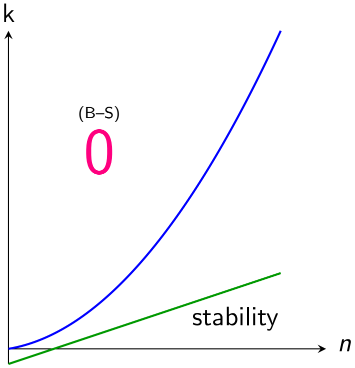

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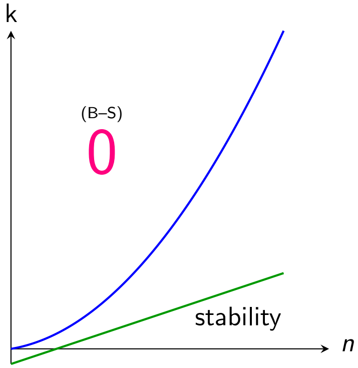

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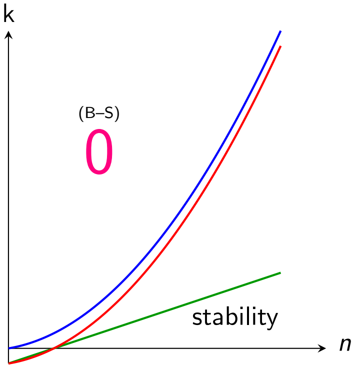

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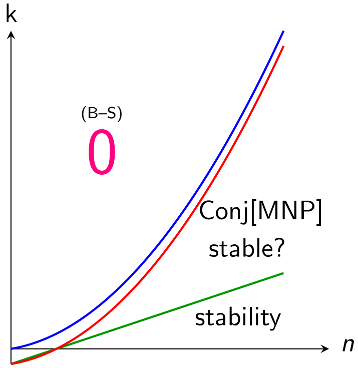

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Main Results

Cohomology of $\Gamma_{0,n}(p)$

$$\Gamma_{0,n}(p) = \left\{ A \in \mathrm{SL}_n(\mathbb{Z}) \;\middle|\; A \equiv \begin{pmatrix} * & * & \cdots & * \\ 0 & * & \cdots & * \\ \vdots & \vdots & \ddots & \vdots \\ 0 & * & \cdots & * \end{pmatrix} \bmod p \right\}$$

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Answer 1:

Borel–Serre 1973: $\mathrm{H}^k(\Gamma_{0,n}(p); \mathbb{Q}) \cong 0$ for $k > \binom{n}{2}$.

Answer 2: Main Theorem

Theorem [A.]

For $k = \binom{n}{2}$,

$$\mathrm{H}^{\binom{n}{2}}(\Gamma_{0,n}(p); \mathbb{Q}) \cong 0 \text{ if } p \in \{2, 3, 5, 7, 13\} \text{ or } p \leq 6n - 14$$

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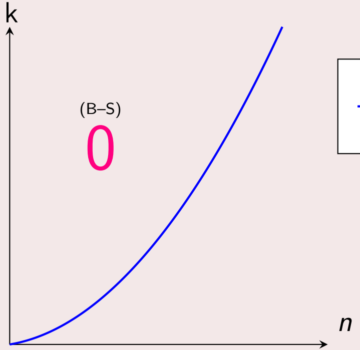

— $k = \binom{n}{2}, H^k(\Gamma_{0,n}(p); \mathbb{Q}) = 0$ for $p \in \{2, 3, 5, 7, 13\}$ or $n \geq \frac{p+14}{6}$

Fig 3: $H^k(\Gamma_{0,n}(p); \mathbb{Q})$

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n \ p	2	3	5	7	11	13	17	19	23
6	0	0	0	0	0	0	0	0	
5	0	0	0	0	0	0	0	0	
4	0	0	0	0	$\mathbb{Q}$	0	$\mathbb{Q}$	$\mathbb{Q}$	
3	0	0	0	0	$\mathbb{Q}^2$	0	$\mathbb{Q}^2$	$\mathbb{Q}^2$	
2	$\mathbb{Q}$	$\mathbb{Q}$	$\mathbb{Q}$	$\mathbb{Q}$	$\mathbb{Q}^3$	$\mathbb{Q}$	$\mathbb{Q}^3$	$\mathbb{Q}^3$	$\mathbb{Q}^4$
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Proof Sketch

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$$\mathrm{H}^{\binom{n}{2}}(\Gamma_{0,n}(p);\mathbb{Q}) \overset{B-S}{\cong} \mathrm{H}_0(\Gamma_{0,n}(p);\mathrm{St}_n) \overset{\text{def}}{=} (\mathrm{St}_n)_{\Gamma_{0,n}(p)},$$

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where $G \curvearrowright M$, $M_G := M/\langle g\cdot m - m \mid g \in G,\ m \in M\rangle$.

Steinberg Module

Definition

The **Tits building** $\mathcal{T}_n$ is a simplicial complex where

- vertices: $0 \subsetneq V \subsetneq \mathbb{Q}^n$,
- k-simplices: $0 \subsetneq V_0 \subsetneq \cdots \subsetneq V_k \subsetneq \mathbb{Q}^n$.

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The Steinberg module $\mathrm{St}_n := \widetilde{\mathrm{H}}_{n-2}(\mathcal{T}_n; \mathbb{Q})$.

$$\mathrm{SL}_n(\mathbb{Z}) \curvearrowright \mathcal{T}_n \quad \rightsquigarrow \quad \mathrm{SL}_n(\mathbb{Z}) \curvearrowright \mathrm{St}_n .$$

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We want to compute

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The way to approach $(\mathrm{St}_n)_{\Gamma_{0,n}(p)}$:

$$\mathrm{St}_n := \widetilde{\mathrm{H}}_{n-2}(\mathcal{T}_n;\mathbb{Q}) \longrightarrow \widetilde{\mathrm{H}}_{n-2}(\mathcal{T}_n/\Gamma_{0,n}(p);\mathbb{Q})$$

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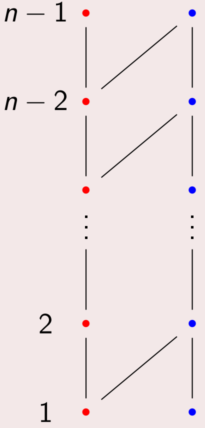

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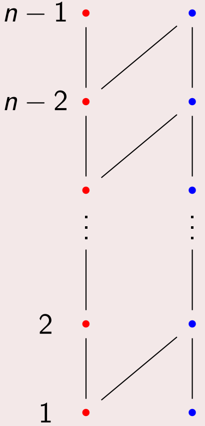

Red Vertex $\leftrightarrow$ subspace *doesn't* contain the basis vector $e_1 \mod p$

Blue Vertex $\leftrightarrow$ subspace *does* contain the basis vector $e_1 \mod p$

Edges indicate containment

Proposition [A.]

$\mathcal{T}_n/\Gamma_{0,n}(p)$ is contractible when $n \geq 3$, and a discrete set of 2 points if $n = 2$. In particular:

$$\widetilde{\mathrm{H}}_{n-2}(\mathcal{T}_n/\Gamma_{0,n}(p); \mathbb{Q}) = \begin{cases} 0 & n \geq 3, \\ \mathbb{Q} & n = 2 \end{cases}$$

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Main Theorem reformulated

- The map Ψ is an isomorphism if $p \in \{2,3,5,7,13\}$ or $p \leq 6n-14$
- Ψ is *not* an isomorphism if $n \in \{2,3\}$ and $p \notin \{2,3,5,7,13\}$

When $n=2$, the rank of $\mathrm{H}^1(\Gamma_{0,2}(p);\mathbb{Q})$ is fully determined by the class of $p \mod 12$

Homological/Rep. stability perspective

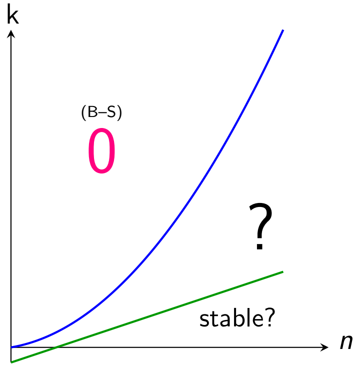

Fig 4: $\mathrm{H}^k(\Gamma_{0,n}(p);\mathbb{Q})$

— $k = \binom{n}{2}$, $\mathrm{H}^k(\Gamma_{0,n}(p);\mathbb{Q}) = 0$ for $n \gg p$